International Journal of Theoretical Physics, Vol. 13, No. 3 (1975), pp. 153–173.

Some Theoretical and Experimental Aspects of the Tachyon Problem

DIETMAR KIRCH

D-5100 Aachen, Kasinostr. 102, Federal Republic of Germany

Received: 16 July 1974

Abstract

In this paper, the hypothetical existence of tachyons is re-examined. It has previously been doubted whether tachyons would exist, for it can be shown with the space-time diagram used in the Special Theory of Relativity that they are moving backward in time for some moving systems, and this would contradict the principle of causality.

Some scientists tried to save the tachyon theory by using the so-called reinterpretation principle, but it is considered that this can be easily refuted.

This paper shows that it can be concluded from the Einstein–Planck frequency relation and the de Broglie wave equation that the absolute value of the mass of the tachyons must be much lower than the rest mass of the electrons—nearly zero. But that means that they do not have any measurable energy if they move with a finite value about c. But if tachyons with $v > c$ cannot be measured they cannot cause causality paradoxes.

The low mass of the tachyons has also consequences for experimental search, and it will be shown that the previous search was doomed to failure.

The limits of the photo-production cross-section for tachyons in lead can be given as $1.5 \cdot 10^{-32} \leqslant \sigma < 7 \cdot 10^{-31}$ cm^2.

Further, it can be shown that the kinetic energy of the tachyons is identical to their total energy.

We see that tachyons cannot be regarded as 'faster than light particles'—they are more a new kind of luxons. If they were to be uncharged it would be nearly impossible to distinguish them from this class of particles.

1. *Introduction*

It has previously been concluded from the Special Theory of Relativity that speeds faster than the speed of light are impossible. Einstein wrote in 1905 that speeds above that of light have no possibility of existence. For decades there has been no reason to doubt this conclusion.

However, it has recently been stated by Bilaniuk *et al.* (1962 and later) and Feinberg (1967) that the contrary is true: the Theory of Relativity makes possible the existence of bodies with such speeds and is capable of describing some of their particular characteristics.

Two classes of bodies are known at present: Class I, the tardyons. These are the well-known elementary particles such as nucleons and electrons. Class II, the luxons. This group contains the particles which move at the speed of light, such as photons, neutrinos, and gravitons.

But what characteristics would the particles of class III, the tachyons, have? And above all, how does one reach the conclusion that these bodies are compatible with the Theory of Relativity? In the strictest sense, the Special Theory of Relativity shows only that a subluminal body in one inertial system cannot have a superluminal velocity in another inertial system, provided that the second inertial system is not moving faster than the speed of light relative to the first inertial system. Furthermore, it can be shown that a body moving at less than

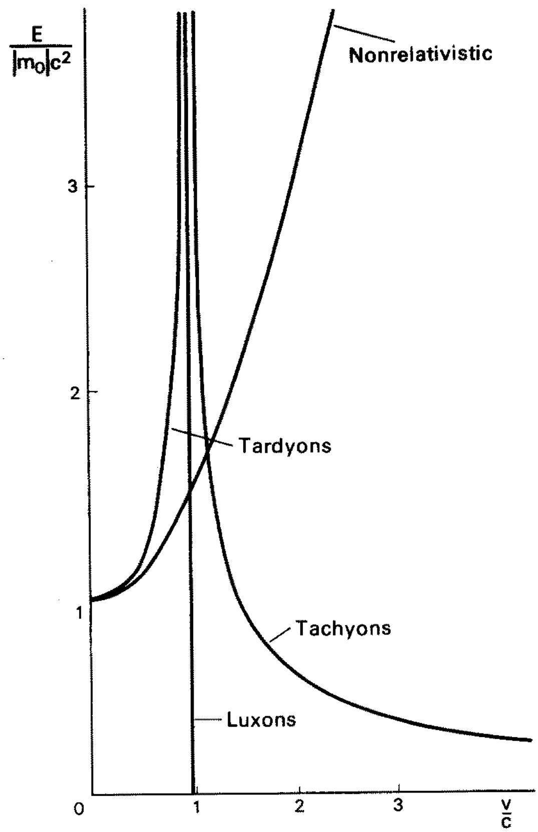

Figure 1.—The energy of tardyons increases as they accelerate toward c. The luxon curve is a straight line, since their velocity is c, regardless of their energy. The energy of tachyons increases as they are decelerated toward c. At infinite velocities, their energy is zero. The non-relativistic curve is added to show that at low speeds the classical formula $E = P^2/2m_0$ coincides with the relativistic formula.

the speed of light cannot be accelerated to exceed the speed of light with a finite expenditure of energy. Assuming that tachyons have a speed exceeding the speed of light at all times and in every inertial system, there are no contradictions to the Special Theory of Relativity.

This can be expressed by Einstein's equation if an imaginary rest mass is introduced:

$$E = m_0c^2/(1 - v^2/c^2)^{1/2} \qquad \text{with} \qquad m_0 = im_* \qquad E = m_*c^2/(v^2/c^2 - 1)^{1/2}$$

This equation yields to the energy diagram shown in Fig. 1. From this diagram it is obvious that speaking of an imaginary mass of tachyons is meaningless.

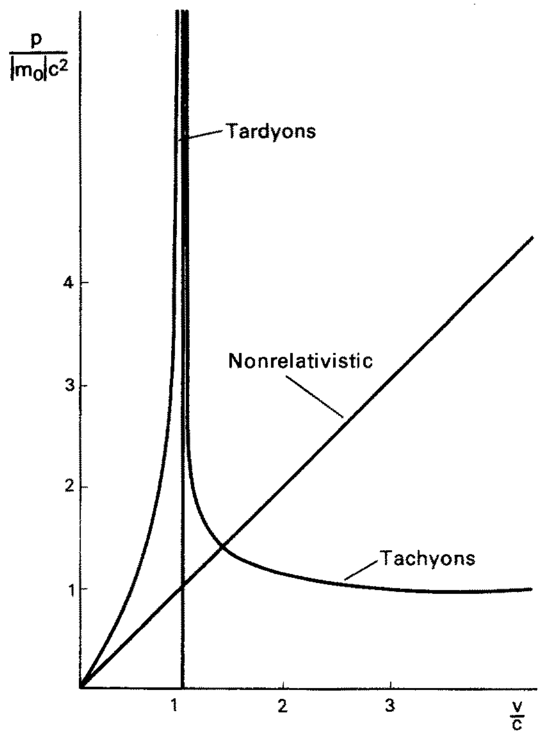

Figure 2.—Impulse diagram; when the speed of tachyons approaches infinity their impulse has the value $p = m_*c$.

The mass is not imaginary as long as the tachyons move in excess of the speed of light. Therefore, tachyons must move at such speeds at all times; nothing else is possible. With a finite energy expenditure a tachyon can no more be decelerated to a speed less than the speed of light than a tardyon can be accelerated to a speed exceeding that of light. (In this case, the tardyon would have an imaginary mass.)

However, one problem causes particular difficulties: the question of causality. As seen in Fig. 3, bodies moving faster than the speed of light are characterised by a backward motion in time relative to some moving systems. What consequences can this have?

Stipulate a machine A to which an inertial system S relates, and a machine A' to which an inertial system S' relates, moving with a speed w relative to S. A is so programmed that at time t_1 it sends a signal to A' by means of a tachyon, but if A receives a tachyon at any time then it cannot send out any more

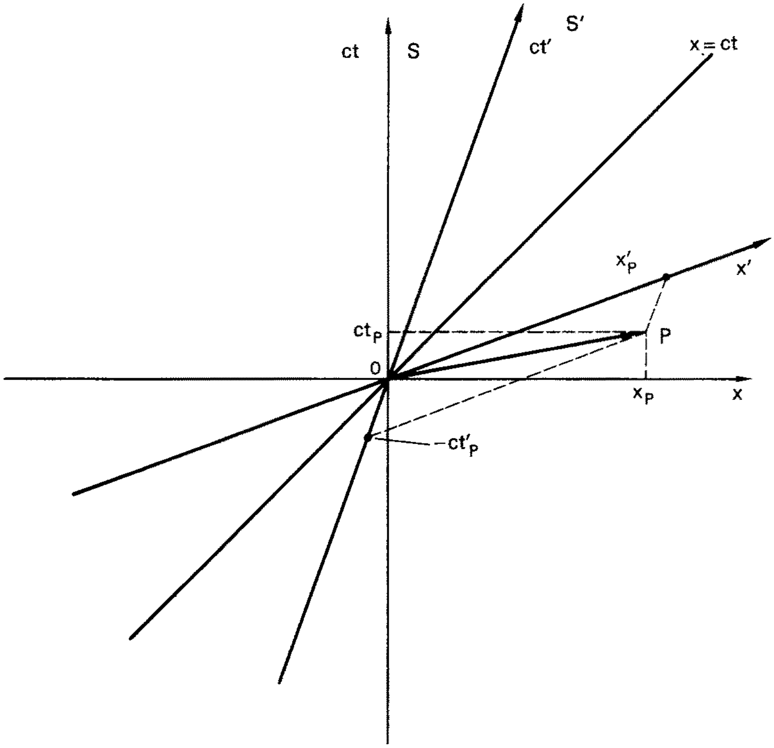

Figure 3.—A tachyon moves backward in time relative to a moving system S' (negative time coordinate $-ct'_P$).

tachyons after that time. A' is programmed to send a tachyon to A whenever it receives one.

At the time t_1, A sends a tachyon to A'. The tachyon arrives at A' at the time t'_2 before t'_1 (seen from S'). According to its programme, A' sends a tachyon to A which arrives at the time t'_3 before t'_2 (seen from S'). Since t_3 precedes t_1 (seen from S and S' because of the Law of Transitivity),† A cannot send a tachyon at the time t_1. However A would not have received a tachyon at the time t_3, so that it must send a tachyon at the time t_1. . . . This is the case which can be described by the term 'causal loop'. To avoid such paradoxes of causality, Bilaniuk & Sudarshan (1969) have simply reversed the direction of the four vector for the moving system S' (see Figs. 4 and 5). These authors

† Seen from S: $t_2 > t_1 > t_3$; seen from S': $t'_1 > t'_2 > t'_3$.

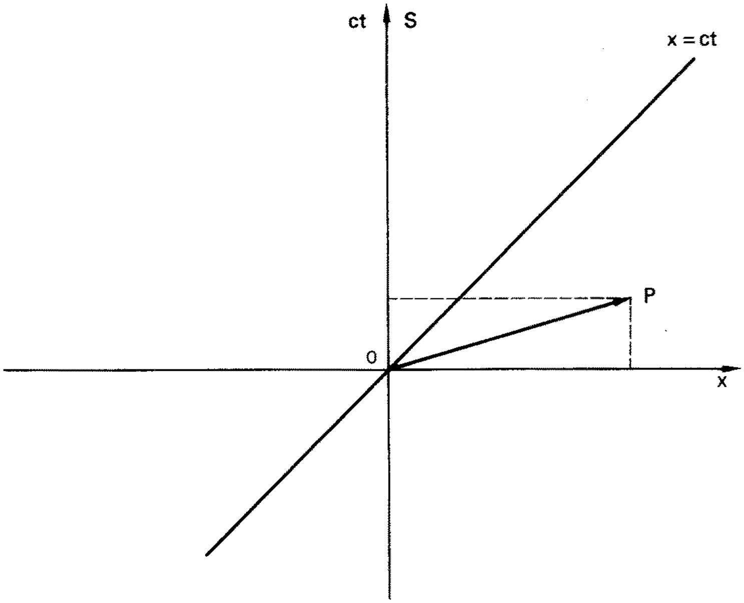

Figure 4.—Seen from S: a tachyon is first emitted in O, and then absorbed in P.

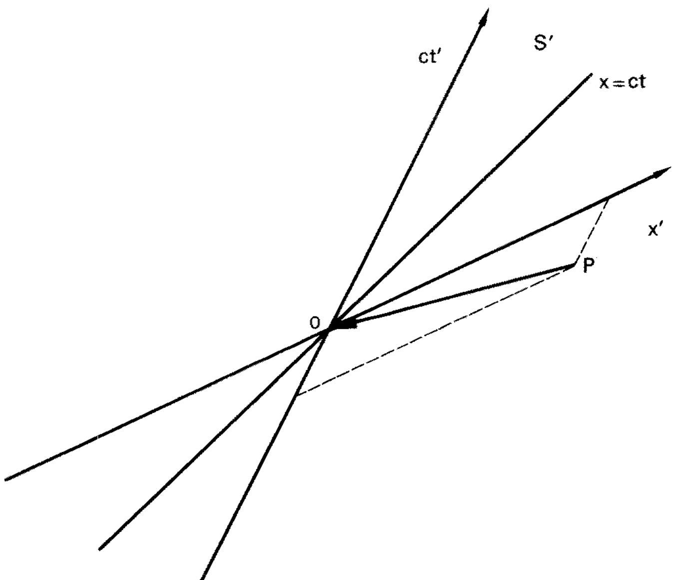

Figure 5.—Seen from S': a tachyon emitted in P and then absorbed in O. Emitter and absorber are exchanged but the law of cause and effect appears to be salvaged.

then say that, seen from the resting system S, the tachyon T is sent out at the space-time point O and then received at P. Seen from S', T is first sent out at P and then received at O. They call this the reinterpretation principle. However, this does not solve the causality problem, as can easily be shown.

2.

2.1. *The Reinterpretation Principle is not Applicable (see Figs. 6 and 7)*

Stipulate again a machine A to which an inertial system S relates, and a machine A' to which an inertial system S' relates, moving with velocity w relative to S. The two machines are programmed as in the previous example.

At the time t_1 A sends out a tachyon T. It arrives at A' at the time t'_2. Due to the exchange principle (reversal of the four vectors causes an exchange of emitter and absorber), A' appears to have sent the tachyon (seen from S'). In accordance with its programme, A' is only capable of sending a tachyon if it has previously received one. Therefore, as viewed from S', A must send out a tachyon at the time t'_3 prior to t'_2. Because of the exchange principle A then appears, seen from system S, to have received a tachyon at the time t_3 prior to t_1. Then, however, A cannot send a tachyon at time t_1, and we have the same paradox as before. We can also have a paradox (but not causal), if we simplify our arrangement.

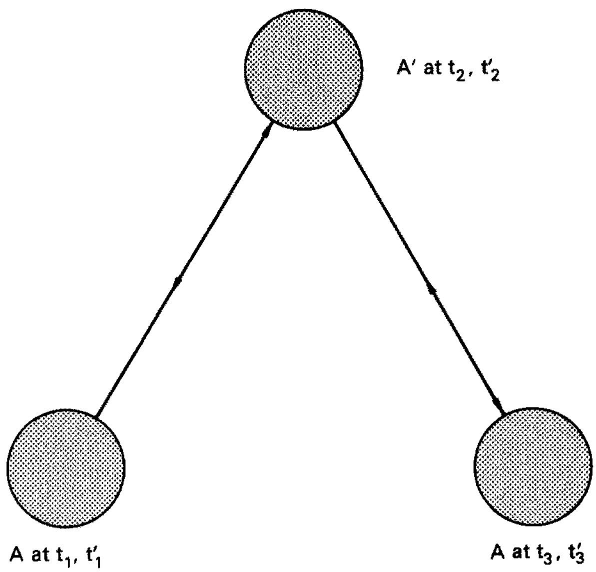

Figure 6.—Respective space-time system S and S' are assigned to A and A'; from these systems the events can be observed. Naturally, the event can be observed from other systems also; this, however, is not of interest for our considerations. Points of arrows at the ends of the vectors: process seen from S. Points of arrows in the middle of the vectors: process seen from S'.

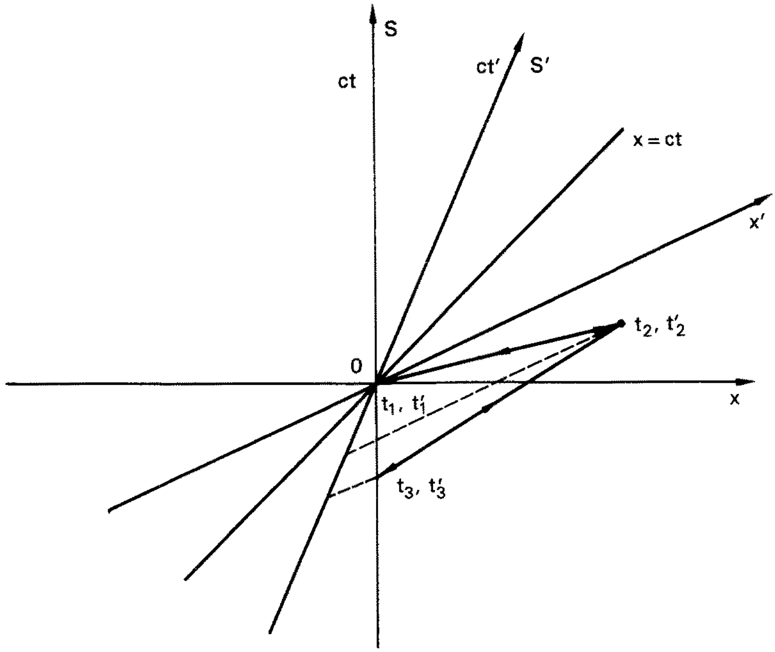

Figure 7.– Points of arrows at the ends of the vectors: process seen from S. Points of arrows in the middle of the vectors: process seen from S'.

We must only construct A so that it can only send out tachyons, and A' so that it can only detect tachyons. We see: how in this case, the observer in S' can mean that A' at time t'_2 was able to send out a tachyon?

The reinterpretation principle has yet another weakness, as Bilaniuk & Sudarshan (1969) have recognised: 'Nonetheless some questions as to the invariance of the energy-momentum four-vector remain'. That is, one cannot simply change the direction of a vector at will. Also, the conclusions of these authors have been criticised for reasons of quantum mechanics (Newton, 1967).

2.2. *The Kinetic Energy of the Tachyons*

To compute the kinetic energy of the tachyons, using the definition of momentum, we use the same procedure as for computing the kinetic energy of the tardyons in the classical Theory of Special Relativity. That is, recalling that $v = ds/dt$, we obtain

$$E_k = \int_a^b F_T\, ds = \int_a^b d/dt\bar{\gamma} m_* v\, ds \;\dagger \qquad F_T = \text{tangential force}$$

$$= \int_a^b v\, d(\bar{\gamma} m_* v) = \bar{\gamma} m_* v^2 - \int_a^b \bar{\gamma} m_* v\, dv = \bar{\gamma} m_* v^2 - m_* c^2 \bar{\gamma}^{-1}\Big|_a^b$$

† Notation: $\bar{\gamma} = i\gamma = 1/(v^2/c^2 - 1)^{1/2}$.

We have two possibilities for integrating this expression: 1) from $a = v$ to $b = \infty$, 2) from $a = c$ to $b = v$. In the first possibility we have the result $-\infty$ for $c < v < \infty$, and $+\infty$ for $v = b = \infty$. But because the total energy of the tachyons is 0 when we have $v = \infty$ their kinetic energy cannot be $+\infty$. We can therefore exclude this possibility.

In the second possibility we obtain $E_k = E$. We obtain the same result if we integrate without integration limits and set the integration constant = 0.

It is clear that the second possibility is to be preferred. We can also say that the total energy of the tachyons is identical to their kinetic energy. It seems, therefore, to be a principle in nature that to receive the kinetic energy of a particle it must be integrated from the state of minimum movement to the state of movement in which the particle is moving.

2.3. *Further Examination of the Causality Problem*

The causality problem arises only in a certain velocity region. Suppose that a tachyon is moving at a speed v relative to an inertial system S, and an inertial system S' moves with a speed w relative to S. The tachyon passes under the x'-axis only when $vw > c^2$. But this condition will give us the solution of the causality problem.

From the Einstein–Planck relations it follows that

$$E = \bar{\gamma} m_* c^2 = \hbar\omega \qquad \omega = \bar{\gamma} m_* c^2/\hbar \qquad \lim_{v\to\infty} \omega = 0 \qquad (2.3.1)$$

and from the de Broglie relation follows that

$$k = p/\hbar = \bar{\gamma} m_* v/\hbar \qquad \lim_{v\to\infty} k = m_* c/\hbar.$$

If we now assume, for example, that m_* has a value identical to the rest mass m_{e0} of an electron, we see that k has a finite value for $v \to \infty$, but $\omega \to 0$ for $v \to \infty$.

But this is contradictory, because we cannot have a finite number of waves and a frequency which is identical to zero at the same time. But if we assume that $m_* \ll m_{e0}$, nearly 0, we see that for $v \to \infty$ k has an extremely low value, nearly 0, therefore ω must also have a value near 0. But this agrees with equation (2.3.1). We can therefore conclude that tachyons must have an extremely low mass. But if m_* is much less it follows that if they have measurable energy and momentum they must have a velocity near c. Tachyons with velocities less than c can never be detected because of their zero energy. That means that our condition $vw > c^2$ can never be satisfied for measurable tachyons, causality paradoxes are therefore excluded.

In our example with A and A' it would also be impossible for A to send out a zero energy tachyon. Such a process would be spontaneous because of the zero energy input. But this process is probably limited because the tachyons are supposed to obey Fermi statistics (Feinberg, 1967). Those energy states which it is possible to reach by spontaneous creation are most probably already filled.

2.4. *The Time and Length of Tachyons*

We now discuss the relativistic time and length of tachyons. Equations (2.4.1) and (2.4.2) show that, with increasing velocity, the passage of time for the tachyons and their length increase. With regard to time, it is notable how well this harmonises with our previous knowledge of time: for resting bodies time passes 'normally', for moving bodies the passing of time is slowed, at the speed of light time stands still, and for superluminal bodies the passing of time is increased.

$$t = \gamma t' \qquad \text{with} \qquad t = t_* i$$
$$t' = t_*(v^2/c^2 - 1)^{1/2} \lim_{v\to\infty} t' = \infty \tag{2.4.1}$$

$$l' = \gamma l \qquad \text{with} \qquad l = l_* i$$
$$l_* = l'(v^2/c^2 - 1)^{1/2} \lim_{v\to\infty} l_* = \infty \tag{2.4.2}$$

2.5. *The Electric and Magnetic Fields of the Tachyons*

Before we can calculate the electric and magnetic fields, we have to find an expression for the force. The force acting on a particle as measured from S and S' is, respectively,

$$\vec{F} = d\vec{p}/dt \qquad \text{and} \qquad \vec{F}' = d\vec{p}'/dt'$$

as required by the principle of relativity, since both observers must use the same equations of motion. We shall compute this relation only for the special case in which the tachyon is momentarily at rest in the system S'.

We then obtain $F'_{x'} = dp'_{x'}/dt'$. From the Lorentz transformation it follows that

$$p'_{x'} = (p_x - wE/c^2)/(1 - w^2/c^2)^{1/2} \qquad \text{with} \qquad p'_{x'} = ip'_{x'*}$$
$$ip'_{x'*} = -i^2(p_x - wE/c^2)/(1 - w^2/c^2)^{1/2} \quad \text{with} \qquad m_0 = im_*$$
$$p'_{x'*} = (\bar{\gamma} m_* w - \bar{\gamma} m_* v)/(w^2/c^2 - 1)^{1/2} = (Ew/c^2 - p_x)/(w^2/c^2 - 1)^{1/2}$$

If now $v = w$, we have $p'_{x'*} = 0$, showing that this expression is logically consistent.

The corresponding expression for energy is

$$E'_* = \bar{\gamma} m_*(vw - c^2)/(w^2/c^2 - 1)^{1/2}$$

We therefore obtain, with $v = w$,

$$\begin{aligned} F'_{x'} &= (dt/dt')(d/dt)((Ev/c^2 - p_x)/(v^2/c^2 - 1)^{1/2}) \\ &= (dt/dt')\bar{\gamma}((v/c^2)(dE/dt) - dp_x/dt) \end{aligned} \tag{2.5.1}$$

Now from the Lorentz transformation we have that

$$t = \gamma(t' + vx'/c^2) \qquad \text{with} \qquad t = t_* i$$
$$t_* = \bar{\gamma}(t' + vx'/c^2)$$

$$dt_*/dt' = (d/dt')\bar{\gamma}(t' + vx'/c^2)$$
$$= \bar{\gamma}, \qquad \text{because} \qquad dx'/dt = 0$$

Also, according to the definition of force, $dp_x/dt = F_x$. From the definitions of energy E and kinetic energy $E_k = E$, as well as the fact that the work $F_x dx$ must be equal to dE_k, we have that

$$dE/dt = dE_k/dt = F_x dx/dt = F_x v$$

In this case we have $dx/dt = v$ because we have chosen the common axis X and X' parallel to the relative velocity of the observers. Making all these substitutions in equation (2.5.1) we finally obtain

$$F'_{x'} = F_x \tag{2.5.2}$$

For the component parallel to the Y-axis, since $F_y = dp_y/dt$, we obtain

$$F'_{y'} = dp'_{y'*}/dt' = (dt/dt')(d/dt)p'_{y'*}$$
$$= (dt/dt')(dp_y/dt) = \bar{\gamma}F_y \tag{2.5.3}$$

and parallel to this

$$F'_{z'} = \bar{\gamma}F_z \tag{2.5.4}$$

Suppose that at rest, relative to S', there are two charges q and Q (Fig. 8). The two charges are then in motion relative to S. The value of the two charges is the same for the observers in S and S'. To the observer in S' there is only an electric interaction between Q and q, and the force measured on q is $\vec{F}' = q\vec{\mathscr{E}}'$, where $\vec{\mathscr{E}}'$ is the electric field produced by Q at q, as measured in S'.

On the other hand, since from S the charge Q is seen in motion, the observer

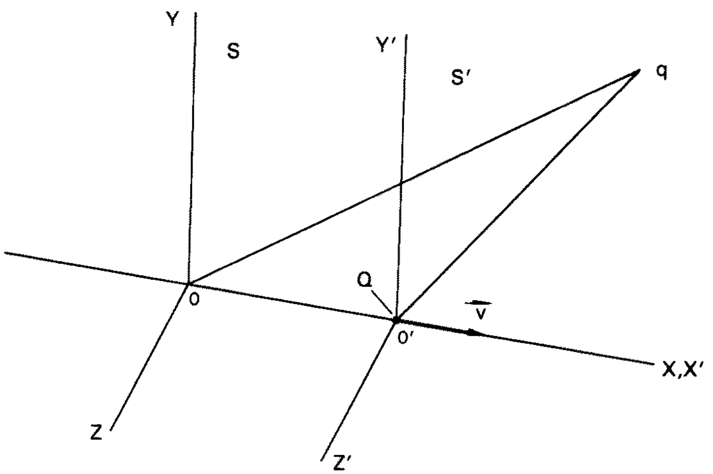

Figure 8.—Comparison of electromagnetic measurements by two observers in relative motion.

notices that Q produces an electric field $\vec{\mathscr{E}}$ and a magnetic field $\vec{B}$, and since q is also observed to be in motion with velocity $\vec{v}$, the force exerted by Q on q that the observer in S measures is

$$\vec{F} = q(\vec{\mathscr{E}} + \vec{v} \times \vec{B})$$

Therefore the components of $\vec{F}$ relative to frame XYZ are

$$F_x = q\mathscr{E}_x, \qquad F_y = q(\mathscr{E}_y - vB_z), \qquad F_z = q(\mathscr{E}_z + vB_y) \qquad (2.5.5)$$

The components of $\vec{F}'$ relative to frame $X'Y'Z'$ are

$$F'_{x'} = q\mathscr{E}'_{x'}, \qquad F'_{y'} = q\mathscr{E}'_{y'}, \qquad F'_{z'} = q\mathscr{E}'_{z'} \qquad (2.5.6)$$

Substituting the values of the components given by equations (2.5.5) and (2.5.6) in those given by equations (2.5.2), (2.5.3) and (2.5.4), and cancelling the common factor q, we obtain

$$\mathscr{E}_x = \mathscr{E}'_{x'}, \qquad \mathscr{E}'_{y'} = \bar{\gamma}(\mathscr{E}_y - vB_z), \qquad \mathscr{E}'_{z'} = \bar{\gamma}(\mathscr{E}_z + vB_y) \qquad (2.5.7)$$

The inverse transformations of equation (2.5.7) are obtained by exchanging the fields and reversing the sign of v, since frame XYZ moves with the velocity $-v$ relative to $X'Y'Z'$. Thus, if the observer in S' measures an electric field $\vec{\mathscr{E}}'$ and a magnetic field $\vec{B}'$, the electric field measured in S is given

$$\mathscr{E}_x = \mathscr{E}'_{x'}, \qquad \mathscr{E}_y = \bar{\gamma}(\mathscr{E}'_{y'} + vB'_{z'}), \qquad \mathscr{E}_z = \bar{\gamma}(\mathscr{E}'_{z'} - vB'_{y'}) \qquad (2.5.8)$$

If the charge Q, instead of being at rest in O', is also moving relative to S', then the observer in S' notes a magnetic field $\vec{B}'$ in addition to the electric field $\mathscr{E}'$. A similar but more laborious calculation then gives

$$B'_{x'} = B_x, \qquad B'_{y'} = \bar{\gamma}(B_y + v\,\mathscr{E}_z/c^2), \qquad B'_{z'} = \bar{\gamma}(B_z - v\,\mathscr{E}_y/c^2) \qquad (2.5.8a)$$

We may obtain the inverse transformation of equation (2.5.8a) by exchanging the fields and replacing v by $-v$, resulting in

$$B_x = B'_{x'}, \qquad B_y = \bar{\gamma}(B'_{y'} - v\,\mathscr{E}'_{z'}/c^2), \qquad B_z = \bar{\gamma}(B'_{z'} + v\,\mathscr{E}'_{y'}/c^2) \qquad (2.5.8b)$$

But if Q is at rest relative to S' the observer in S' does not measure any magnetic field, but only an electric field; therefore $B'_{x'} = B'_{y'} = B'_{z'} = 0$. Then the electric field transformations of equation (2.5.8) yield

$$\mathscr{E}_x = \mathscr{E}'_{x'}, \qquad \mathscr{E}_y = \bar{\gamma}\mathscr{E}'_{y'}, \qquad \mathscr{E}_z = \bar{\gamma}\mathscr{E}'_{z'} \qquad (2.5.9)$$

Noting from Fig. 9 that $\vec{\mathscr{E}}'$ makes an angle θ' with $O'X'$ and that $\cos\theta' = x'/r'$, $\sin\theta' = y'/r'$, we have that the components of $\vec{\mathscr{E}}'$ are

$$\mathscr{E}'_{x'} = \mathscr{E}' \cos\theta' = (q/4\pi\epsilon_0)(x'/r'^3)$$

$$\mathscr{E}'_{y'} = \mathscr{E}' \sin\theta' = (q/4\pi\epsilon_0)(y'/r'^3)$$

Using equation (2.5.9) and the fact that, according to the Lorentz transformation, $x_* = x'(v^2/c^2 - 1)^{1/2}$ and $y = y' = y_*$, we may write the components of the field $\vec{\mathscr{E}}$ observed from S as

$$\mathscr{E}_x = \mathscr{E}'_{x'} = (q/4\pi\epsilon_0)(\bar{\gamma}x_*/r'^3)$$
$$\mathscr{E}_y = \bar{\gamma}\mathscr{E}'_{y'} = (q/4\pi\epsilon_0)(\bar{\gamma}y_*/r'^3)$$

We may write this in vector notation as

$$\vec{\mathscr{E}} = (q/4\pi\epsilon_0)(\bar{\gamma}/r'^3)\vec{r} \qquad (2.5.10)$$

showing that the field $\vec{\mathscr{E}}$ is along the radial direction in the XYZ frame. Now

$$r'^2 = x'^2 + y'^2 = x_*^2/(v^2/c^2 - 1) + y_*^2$$
$$= (x_*^2 + y_*^2(v^2/c^2 - 1))/(v^2/c^2 - 1)$$

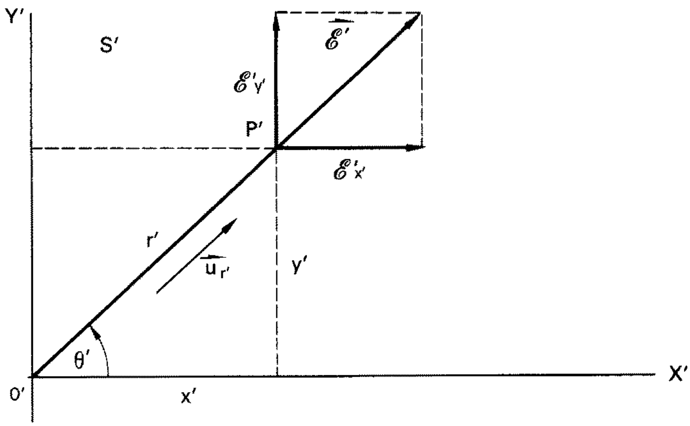

Figure 9.

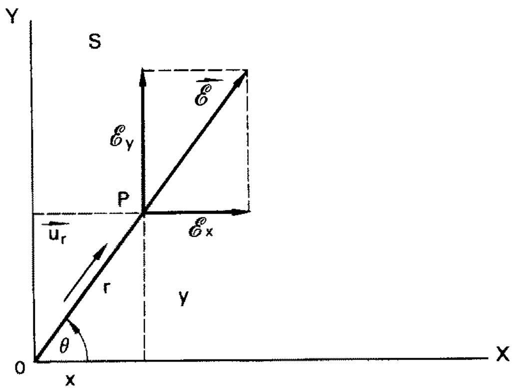

Figure 10.

Figures 9 and 10.—Relativistic transformation of the components of the electric field produced by a charge q at rest relative to S' and located at O'.

and $y_*^2 = r^2 \sin^2\theta$. Therefore

$$r' = \bar{\gamma}((x_*^2 + r^2 \sin^2\theta(v^2/c^2 - 1))^{1/2}$$

Using this relation to eliminate r' in equation (2.5.10), we finally obtain

$$\vec{\mathscr{E}} = (q/4\pi\epsilon_0)((v^2/c^2 - 1)\vec{r}/r^3(\cos^2\theta + \sin^2\theta(v^2/c^2 - 1))^{3/2})$$
$$\vec{\mathscr{E}} = (q/4\pi\epsilon_0 r^2)((v^2/c^2 - 1)\vec{u}_r/(\cos^2\theta + \sin^2\theta(v^2/c^2 - 1))^{3/2}) \qquad (2.5.11)$$

A very convincing result, because, if $v = c(2)^{1/2}$, we have a field which is identical to a field of a tardyon at rest; this is parallel to the behaviour of the mass of the tachyons. At this velocity, the mass is identical to the proper mass $(\bar{\gamma}(c(2)^{1/2})m_* = m_*)$. Applying the relation $\vec{B} = \vec{v} \times \vec{\mathscr{E}}/c^2$, which is of general validity, and using equation (2.5.11), we find the magnetic field of a charged tachyon as

$$\vec{B} = (\mu_0 q/4\pi r^2)((v^2/c^2 - 1)(\vec{v} \times \vec{u}_r)/(\cos^2\theta + \sin^2\theta(v^2/c^2 - 1))^{3/2})$$

To find an expression for θ, we obtain from equation (2.5.11)

$$\theta = \arcsin\,(((q\vec{u}_r/4\pi\epsilon_0\vec{\mathscr{E}}\bar{\gamma}r^2)^{2/3} - 1)/(\gamma^{-2} - 1))^{1/2}$$

To simplify this expression we arbitrarily set

$$\vec{\mathscr{E}} = (\mathscr{E}_x, 0, 0) \qquad \text{and} \qquad \vec{u}_r = (1, 0, 0)$$

and obtain

$$\theta = \arcsin\,(((q/4\pi\epsilon_0\,\mathscr{E}_x\bar{\gamma}r^2)^{2/3} - 1)/(\bar{\gamma}^{-2} - 1))^{1/2}$$

2.6. *The Magnetic Field of a Rectilinear Current of Charged Tachyons*

Let us consider an infinite row of equally spaced charges which are moving along the X-axis with velocity v relative to S (Fig. 11) and which thus constitutes a rectilinear electric current. If λ is the electric charge per unit length, the electric current measured from S is $I = \lambda v$. Now let us consider a system S' moving in the X-direction with velocity v. Relative to S', the charges appear at rest and, measured from S', there is only an electric field. However, recorded from S, there is an electric and a magnetic field.

The charge in a segment dx (as measured from S) is $dq = \lambda\,dx$. The observer in S' measures the same charge, but, because of the Lorentz contraction, the segment appears to have a length dx' such that $dx_* = (v^2/c^2 - 1)^{1/2}\,dx'$. Therefore the observer in S' measures a different charge per unit length λ' given by

$$\lambda' = \frac{dq}{dx'} = \lambda\frac{dx_*}{dx'} = (v^2/c^2 - 1)^{1/2}\lambda$$

The electric field as measured from S' is transverse. At a point P it is given by $\mathscr{E}' = \lambda'/2\pi\epsilon_0 R'$, which can be shown by elementary calculations. By placing

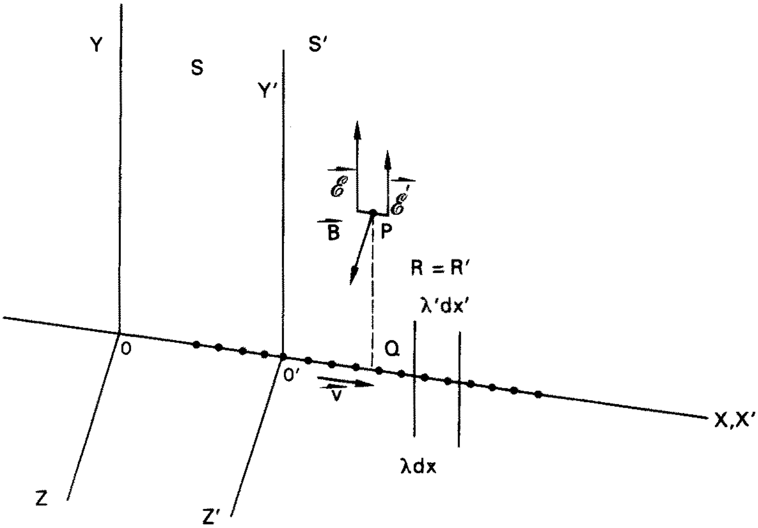

Figure 11.—Electromagnetic field produced by a stream of charges moving along the X-axis as observed from two systems in relative motion.

the Y-axis parallel to the line PQ and noting that $R = R'$ because it is of transverse length, we may write

$$\mathscr{E}'_{x'} = 0, \qquad \mathscr{E}'_{y'} = \frac{\lambda'}{2\pi\epsilon_0 R}, \qquad \mathscr{E}'_{z'} = 0$$

Then, using equations (2.5.8) with $\vec{B}' = 0$, we may write the components of the electric field relative to S as

$$\mathscr{E}_x = \mathscr{E}_z = 0, \qquad \mathscr{E}_y = \bar{\gamma}\mathscr{E}'_{y'} = \bar{\gamma}\frac{\lambda'}{2\pi\epsilon_0 R}$$

$$= \frac{\lambda}{2\pi\epsilon_0 R}$$

Similarly, equations (2.5.8) give the components of the magnetic field relative to S as

$$B_x = B_y = 0, \qquad B_z = \bar{\gamma}(v\mathscr{E}'_{y'}/c^2) = \frac{\bar{\gamma}\lambda' v/c^2}{2\pi\epsilon_0 R} = \frac{\mu_0 I}{2\pi R}$$

where we have used the relation $\epsilon_0\mu_0 = 1/c^2$.

2.7. *The Derivation of an Expression for the Energy Radiated per Unit Time by an Accelerated Tachyon*

Larmor's formula $dE/dt = q^2a^2/6\pi\epsilon_0 c^3$ is strictly correct only when the particle is momentarily at rest relative to an observer. To obtain the value of the energy radiated by the charge as measured by an observer who sees the particle moving with velocity $v > c$, we must simply make a Lorentz transformation of all quantities involved in that expression. Suppose that the tachyon is momentarily at rest relative to an observer O' who uses the frame of reference $X'Y'Z'$. Larmor's formula becomes

$$\frac{dE'}{dt'} = \frac{q^2 a'^2}{6\pi\epsilon_0 c^3}$$

Since $dt_* = \bar{\gamma}\, dt'$ and $dE = \bar{\gamma}\, dE'$ we obtain

$$\frac{dE}{dt} = \frac{q^2 a'^2}{6\pi\epsilon_0 c^3}$$

Therefore we must only derive an expression for a'. The X-component of the acceleration of the particle, as measured from S' moving with v relative to S, is

$$a'_{x'} = \frac{dV'_{x'}}{dt'} = \frac{dV'_{x'}}{dt}\frac{dt}{dt'} \qquad (V = \text{particle velocity})$$

with

$$V'_{x'} = \frac{dx'}{dt'}$$

Because

$$\begin{aligned} dx' &= \bar{\gamma}(dx_* - v\, dt_*) = \bar{\gamma}(V_x - v)\, dt_* \\ dy' &= dy_* \\ dz' &= dz_* \\ dt' &= \bar{\gamma}(dt_* - v\, dx_*/c^2) \\ &= \bar{\gamma}(1 - vV_x/c^2)\, dt_* \end{aligned}$$

we have

$$V'_{x'} = \frac{V_x - v}{1 - vV_x/c^2}$$

Using the value of $V'_{x'}$ and inserting the appropriate derivatives, we have

$$\begin{aligned} a'_{x'} &= \left[\frac{a_x}{1 - vV_x/c^2} + \frac{(V_x - v)va_x/c^2}{(1 - vV_x/c^2)^2}\right] \frac{\left(\dfrac{v^2}{c^2} - 1\right)^{1/2}}{1 - vV_x/c^2} \\ &= a_x \frac{-\left(\dfrac{v^2}{c^2} - 1\right)^{3/2}}{(1 - vV_x/c^2)^3} \end{aligned}$$

At the moment when the particle is at rest relative to S', $V_x = v$ and

$$a'_{x'} = \frac{a_x}{\left(\frac{v^2}{c^2} - 1\right)^{3/2}} = \bar{\gamma}^3 a_x$$

By a similar analysis we find that

$$a'_{y'} = \bar{\gamma}^2 a_y, \qquad a'_{z'} = \bar{\gamma}^2 a_z$$

Now

$$\begin{aligned}
a'^2 &= a'^2_{x'} + a'^2_{y'} + a'^2_{z'} \\
&= \frac{a_x{}^2}{(v^2/c^2 - 1)^3} + \frac{a_y{}^2}{(v^2/c^2 - 1)^2} + \frac{a_z{}^2}{(v^2/c^2 - 1)^2} \\
&= \frac{a_x{}^2 + (a_y{}^2 + a_z{}^2)(v^2/c^2 - 1)}{(v^2/c^2 - 1)^3} \\
&= \frac{a_x{}^2 - a_y{}^2 - a_z{}^2 + v^2(a_y{}^2 + a_z{}^2)/c^2}{(v^2/c^2 - 1)^3}
\end{aligned}$$

But $\vec{v} = \vec{u}_x v$ and $\vec{v} \times \vec{a} = -\vec{u}_y v a_z + \vec{u}_z v a_y$, so that $(\vec{v} \times \vec{a})^2 = v^2(a_y{}^2 + a_z{}^2)$. Therefore,

$$a'^2 = \frac{a_x{}^2\, a_y{}^2 - a_z{}^2 + (\vec{v} \times \vec{a})^2/c^2}{(v^2/c^2 - 1)^3}$$

We receive, therefore,

$$\frac{dE}{dt} = \frac{q^2}{6\pi\epsilon_0 c^3} \frac{a_x{}^2 - a_y{}^2 - a_z{}^2 + (\vec{v} \times \vec{a})^2/c^2}{(v^2/c^2 - 1)^3}$$

If the acceleration is parallel to the velocity and $a_x = a$, $\vec{v} \times \vec{a} = 0$ and the expression reduces to

$$\left(\frac{dE}{dt}\right)_{\parallel} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3 (v^2/c^2 - 1)^3}$$

On the other hand when the acceleration is perpendicular to the velocity $(\vec{v} \times a)^2 = v^2 a^2$ and the expression reduces to

$$\left(\frac{dE}{dt}\right)_{\perp} = \frac{q^2 a^2 (v^2/c^2 + 1)}{(v^2/c^2 - 1)^3} \tag{2.7.1}$$

3.

3.1. *The Experimental Search for Tachyons Using Čerenkov Radiation*

If tachyons are produced in pairs, as discussed (Feinberg, 1967), it should be possible to prove their existence using Čerenkov radiation. The energy loss of a charged particle due to Čerenkov radiation is described as follows (Sommerfeld, 1904; Frank & Tamm, 1937):

$$\frac{dE}{ds} = -\frac{4\pi^2 Z^2 e^2}{c^2} \int \left(1 - \frac{c^2}{v^2 n^2}\right) \nu \, d\nu \tag{3.1.1}$$

where Ze is the charge of the incident particle, e is the electron charge, n is the refraction index, and ν is the frequency of the emitted electromagnetic waves. The frequency range of the integral is dependent in practice on the minimum and maximum sensitivity of the photomultiplier used. The discovery of tachyons by means of Čerenkov radiation has been attempted. This was doomed to failure, because the attempt started with the idea that tachyons are irradiating in a vacuum. However, as we have seen in Section 2.3, tachyons with any large energy move at a speed very near the speed of light, so that the radiation is very nearly zero. Therefore the use of some medium cannot be avoided. This is unfortunate, since the medium introduces interfering tardyon particles.

With a gamma source having an energy of less than 1 MeV, the apparatus illustrated in Fig. 12 can be used. If the irradiated energy is not sufficient an electrical field can be applied to decelerate the tachyons. This process increases the energy.

Energies of more than 1 MeV cannot be used because an arrangement of electric or magnetic fields must be used which can single out the produced electron–positron pairs.

But from equation (2.7.1) we obtain that for tachyons dE/dt is much greater because $v \approx c$. Therefore there would not be any energy to be measured as Čerenkov radiation.

In order to find an expression for the path $S(E)$, equation (3.1.1) must be integrated over the possible frequency region from 0 to E/h, since the actual energy loss of the tachyon and its consequences should be described. It follows that:

$$\begin{aligned} dE/ds &= -Z^2 e^2 E^2 (1/c^2 - 1/v^2 n^2)/2\hbar^2 \qquad \text{with} \qquad v \approx c \\ &= -Z^2 e^2 E^2 (1 - 1/n^2)/2\hbar^2 c^2 \qquad \text{with} \qquad 1 - 1/n^2 = a \\ dE/ds &= -Z^2 e^2 E^2 a/2\hbar^2 c^2 \end{aligned}$$

From the above, it follows for path S that,

$$S = -(2\hbar^2 c^2/Z^2 e^2 a) \int_{E_i}^{E_f} dE/E^2 \qquad E_i = \text{initial energy}, \qquad E_f = \text{final energy}$$

$$S = (2\hbar^2 c^2/Z^2 e^2 a)(1/E_f - 1/E_i)$$

with $Z = 1, n = 1.33$ (water), $E_i = 0.5$ MeV, $E_f = 0.99E_i$: $S = 16$ cm. This means that a tachyon radiates its energy over a satisfactorily long path, which of course simplifies the measurement.

If a tachyon, as described in Section 3.1, moves through an electrical field,

$$dE/ds = -(Z^2 e^2 E^2 a/2\hbar^2 c^2) + Ze \text{ grad } V \qquad V = \text{electrical potential}$$

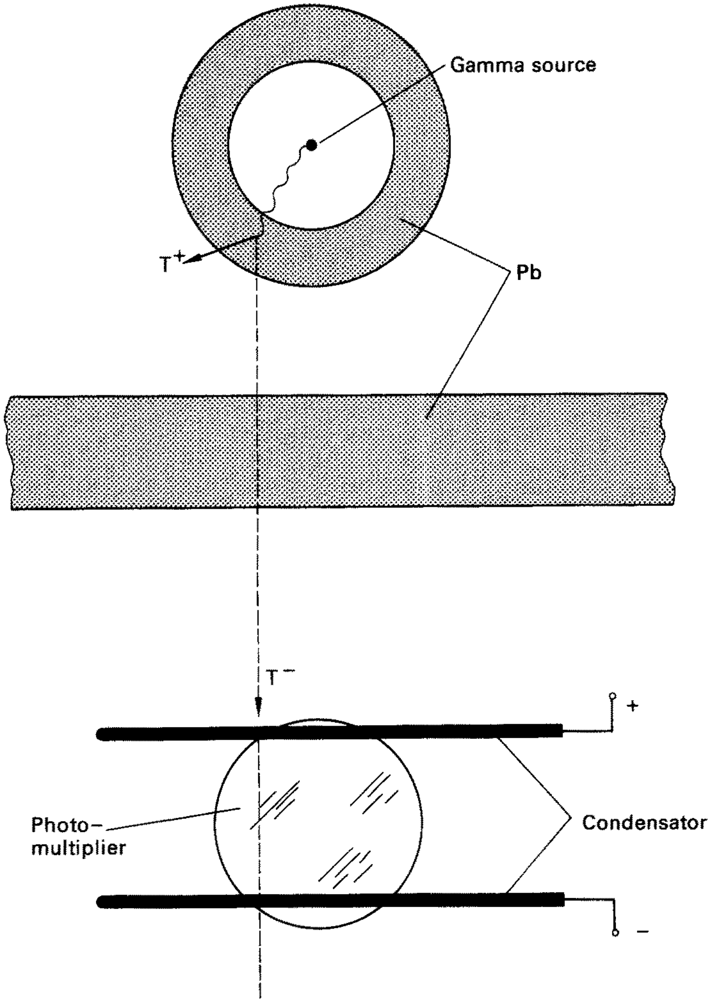

Figure 12.—Schematic diagram of a detector system for charged tachyons for which the energy of the gamma radiation must lie under 1 MeV.

To reach a state of equilibrium, dE/ds must be allowed to be 0, i.e. the tachyon must be allowed to move through the field to regain the energy which it has lost by Čerenkov radiation. From this follows:

$$Z^2e^2E^2a/2\hbar^2c^2 = Ze \text{ grad } V$$
$$\text{grad } V = ZeE^2a/2\hbar^2c^2$$

If of interest, the field needed to maintain a state of equilibrium can be calculated in this way.

Since the possibility that tachyons can be absorbed by tardyons cannot be excluded, their mean free path length is now calculated. In classical terms the condition for the absorption of a tachyon of the charge Ze into an area of the radius R of a charge ze can be given as:

$$Zze^2 = Rvp$$

The geometrical cross-section is then given by

$$\sigma \approx 2\pi R^2 = 2\pi Z^2 z^2 e^4/v^2 p^2 = 2\pi z^2 Z^2 e^4 E^2/(E^2 + m_*^2 c^4)^2$$

Since $m_* c^2$ is negligible, it follows that

$$\sigma \approx 2\pi Z^2 z^2 e^4/E^2$$

$z = Z = 1$ and $E = 2.45 \cdot 10^{-2}$ eV, then

$$\sigma = 3 \cdot 10^{-34}\ \text{m}^2$$

This corresponds to a mean free path length of the tachyon in lead of 10^4 m if the electron is taken as target. If E is larger, the path length increases accordingly. The strength of the lead shield can be made great enough to keep such disturbing influences away from the gamma source.

The expected counting rate of the tachyon is

$$R = \sigma N \phi_\gamma \Omega$$

σ is the photo-production cross-section for tachyons in lead; N is the number of target lead atoms; ϕ_γ is the gamma flux on the lead target; and Ω is the portion of the production solid angle, subtended by the detector, given in per cent. Thereby, Ω is $\Omega(m_*)$. Ω is inversely proportional to m_* and can only be determined by a Monte Carlo calculation. With an estimate of R we can, therefore, put an upper limit on the photo-production cross-section in lead. (Alväger & Kreisler (1968) give R with 0.1 counts/sec for a gamma energy of 0.8 MeV, a flux of $5 \cdot 10^5/\text{cm}^2$ sec and $N = 1.4 \cdot 10^{24}$.)

A Monte Carlo calculation indicates that for $m_* c^2 = 0.1$ MeV Ω is 2%. Because we know that $m_* c^2$ must be much less than 0.1 MeV we obtain for the limits of σ

$$1.5 \cdot 10^{-32} \leqslant \sigma < 7 \cdot 10^{-31}\ \text{cm}^2$$

Since $m_* c^2 \ll E_\gamma$, it follows that tachyons will be produced mainly in the forward direction.

3.2. *Search for Tachyons in Cosmic Rays*

If charged tachyons are existing in the cosmic rays they must react with charged particles as they move through the atmosphere and the earth's outer layers. Then tachyons could be found by the method illustrated by Fig. 13, showing the schematic of a mine 3000 m deep. Every 300 m there is a scintillation counter.

The counters should be connected by an AND switch which should function when the counters are activated with a time increment of 10^{-6} sec. This condi-

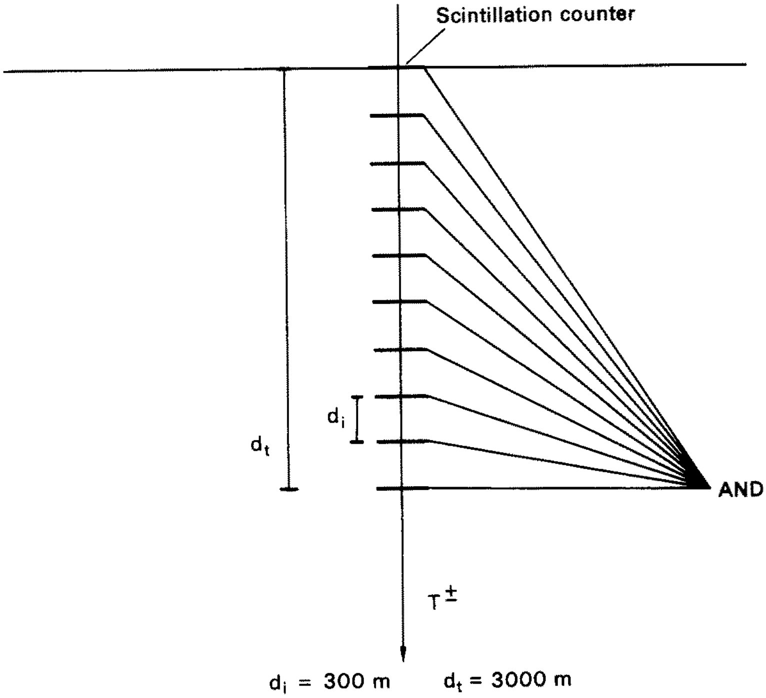

Figure 13.—Schematic diagram of a detection system to prove the existence of tachyons in cosmic rays.

tion would be met by a particle moving past the counters at exactly the speed of light, a condition that only tachyons can fulfil.

4. *Discussion of Results*

It is suggested how the causality problem may be resolved. Nevertheless, its implications are very important for the search for tachyons, as shown in Section 3.1. It is understandable that previous attempts have failed (Alväger & Kreisler, 1968; Murthy, 1971; Clay & Crouch, 1974).

Because of the very low mass of the tachyons and because their total energy is identical to their kinetic energy, it could be seen that they are very closely connected with the luxons and that it would be nearly impossible to distinguish them from the luxons if they should be uncharged.

Further, it could be shown that it seems to be a principle in nature that to receive the kinetic energy of a particle it must be integrated from the state of minimum movement to the state of movement in which the particle is moving.

Acknowledgments

I would like to express my thanks for many stimulating discussions and support to Prof. H. Faissner, Prof. R. Rodenberg, Prof. H. Wagner, Prof. G. Simon, Prof. H.-D. Dietze, Dr. J. v. Krogh, Dr. H. Rechenberg, Dr. Schlereth, H. G. Fasold, H. Geller and M. Liehr. My thanks also to Miss B. Horner and Mrs. L. Gratz for assistance in translation and typing. I also want to thank Mr. Horbach, General Manager of the Rank Xerox office, Aachen, for his valuable assistance.

References

Alväger, T. and Kreisler, M. N. (1968). *Physical Review*, **171**, 1357.
Bilaniuk, O. M. P., Deshpande, V. K. and Sudarshan, E. C. G. (1962). *American Journal of Physics*, **30**, 718.
Bilaniuk, O. M. P. and Sudarshan, E. C. G. (1969). *Physics Today*.
Clay, R. W. and Crouch, P. C. (1974). University of Adelaide (unpublished).
Feinberg, G. (1967). *Physical Review*, **159**, 1089.
Frank, I. and Tamm, I. (1937). *Doklady̆ Akademii nauk SSSR*, **14**, 109.
Murthy, R. (1971). *Lettere al Nuovo Cimento*, **1**(2), 908.
Newton, R. G. (1967). *Physical Review*, **162**, 1274.
Sommerfeld, A. (1904). *Proceedings K. Nederlandse akademic van wetenschappen*, **8**, 346.